4.7 – Inverse Trig Functions (Day 2)

Compositions of Functions

Recall: If $f(x)$ and $f^{-1}(x)$ are truly inverse functions, then for all x values in the domains of $f(x)$ and $f^{-1}(x)$, the following is true:

$f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$

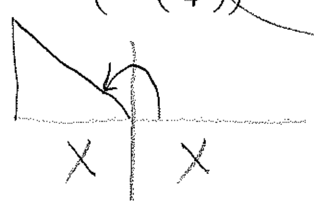
Inverse Properties of Trigonometric Functions:

Always true	**Only true for x-values in the "restricted" domains**
$\sin(\sin^{-1}(x))=x$	$\sin^{-1}(\sin(x))=x$
$\cos(\cos^{-1}(x))=x$	$\cos^{-1}(\cos(x))=x$
$\tan(\tan^{-1}(x))=x$	$\tan^{-1}(\tan(x))=x$

Example 1: Find the exact value of the following:

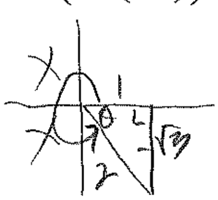
a) $\sin(\sin^{-1}(1))$ $=1$

b) $\cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$



because $\cos^{-1}$ does exist in QII

c) $\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right) = -\frac{\pi}{3}$ or $-60°$

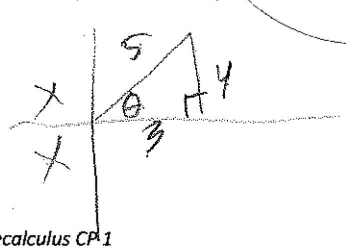


because domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

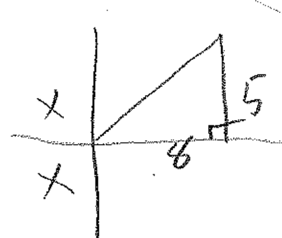
d) $\tan\left(\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\sqrt{3}}{2}$

Example 2: Find the exact value of the following (hint: draw a triangle!):

a) $\sec\left(\arcsin\left(\frac{4}{5}\right)\right) = \frac{5}{3}$



b) $\cot\left(\arctan\left(\frac{5}{8}\right)\right) = \frac{8}{5}$



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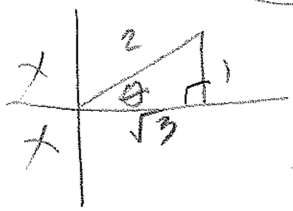
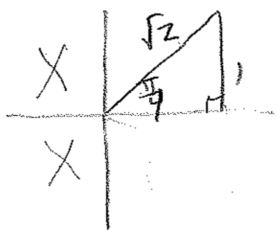
Mixed Practice – You try!

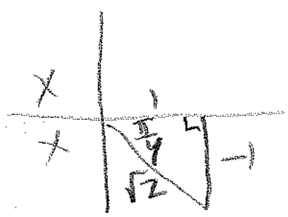
a) $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$



b) $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

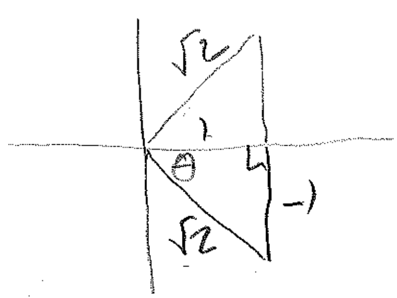


c) $\sin(\tan^{-1}(-1)) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



d) $\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ \text{ or } \frac{\pi}{4}$

must be in QI or QII!



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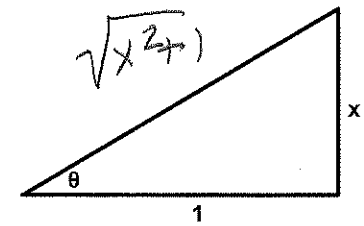
Example 3:



a) Find the hypotenuse of the triangle as a function of x

$$h = \sqrt{1^2 + x^2} = \sqrt{x^2+1}$$

b) Find $\tan\theta$

$$= \frac{x}{1}$$

c) Find $\tan^{-1}(x) = \theta$

$$\theta = \tan^{-1}\left(\frac{x}{1}\right)$$

d) Find the hypotenuse of the triangle as a function of x

e) Find $\sin\left(\underbrace{\tan^{-1}(x)}_{\theta}\right)$ as a ratio involving no trig functions

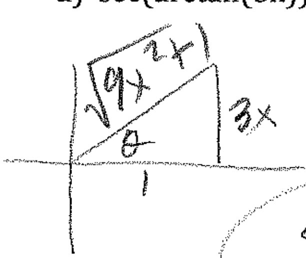
$$\sin\theta = \frac{x}{H} = \frac{x}{\sqrt{x^2+1}}$$

f) Find $\sec\left(\underbrace{\tan^{-1}(x)}_{\theta}\right)$ as a ratio involving no trig functions

$$\sec\theta = \frac{H}{1} = \sqrt{x^2+1}$$

Example 4: Write an algebraic expression that is equivalent to the given expression (hint: draw a triangle!)

a) sec(arctan(3x))

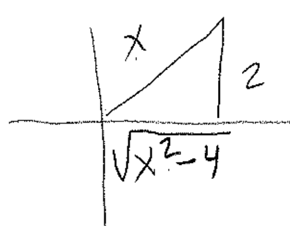


$$(3x)^2 + 1^2 = 9x^2 + 1$$

$$H = \sqrt{9x^2+1}$$

$$\sec\theta = \sqrt{9x^2+1}$$

b) $\cot\left(\arcsin\left(\frac{2}{x}\right)\right) = \frac{\sqrt{x^2-4}}{2}$



Homework: 4.7 Exercises Day 2: p. 349# 37, 39, 41, 49 - 67 odd, 91, 92